

8.1

Model Inverse and Joint Variation

Goal • Use inverse variation and joint variation models.

Your Notes

VOCABULARY

Inverse variation Two variables x and y show inverse variation if they are related as follows: $y = \frac{a}{x}$, $a \neq 0$.

Constant of variation The nonzero constant a in a variation equation

Joint variation When a quantity varies directly with the product of two or more other quantities

INVERSE VARIATION

Two variables x and y show inverse variation if they are related as follows: $y = \frac{a}{x}$, $a \neq 0$

The constant a is the constant of variation, and y is said to vary inversely with x .

Example 1 Classify direct and inverse variation

Tell whether x and y show *direct variation*, *inverse variation*, or *neither*.

Given Equation	Rewritten Equation	Type of Variation
a. $\frac{y}{9} = x$	$y = 9x$	<u>Direct</u> positive correlation
b. $xy = 3$	$y = \frac{3}{x}$	<u>Inverse</u> negative correlation

$Y=3/5$

✓ **Checkpoint** Tell whether x and y show *direct variation*, *inverse variation*, or *neither*.

1. $y = x + 2$ neither	2. $yx = 5$ inverse $y = \frac{5}{x}$	3. $\frac{y}{2.6} = x$ direct $y = 2.6x$
----------------------------------	--	---

x	1	2	4	5
y	5	2.5	1.25	1

x	1	2	3
y	2.6	5.2	7.8

Your Notes

Example 2 Write an inverse variation equation

The variables x and y vary inversely, and $y = 3$ when $x = 6$. Write an equation that relates x and y . Find y when $x = -9$.

$$y = \frac{a}{x} \quad \text{Write general equation for inverse variation.}$$

$$\underline{6} = \frac{a}{\boxed{3}} \quad \text{Substitute for } y \text{ and for } x.$$

$$\underline{18} = a \quad \text{Solve for } a.$$

The inverse variation equation is $y = \frac{18}{x}$. When $x = -9$,
 $y = \frac{18}{-9} = \underline{-2}$.

✓ **Checkpoint** Complete the following exercise.

4. The variables x and y vary inversely, and $y = 4.4$ when $x = 5$. Write an equation that relates x and y . Find y when $x = 8$.

$$y = \frac{22}{x}; 2.75$$

Example 3 Check data for inverse variation

Determine whether m and n show inverse variation. If they do, write a model that gives n as a function of m . Find n when $m = 45$.

m	5	10	15	20	25
n	45	22.5	15	11.25	9

Calculate the product $m \cdot n$ for each data pair in the table.

$$\begin{aligned} 5(45) &= \underline{225} & 10(22.5) &= \underline{225} & 15(15) &= \underline{225} \\ 20(11.25) &= \underline{225} & 25(9) &= \underline{225} \end{aligned}$$

Each product is equal to 225. So, the data do show inverse variation. A model relating m and n is

$$m \cdot n = \underline{225} \text{ or } n = \frac{225}{m}.$$

The value of n when $m = 45$ is $n = \frac{225}{45} = \underline{5}$.

Your Notes

- ✓ **Checkpoint** Do the data below show inverse variation? If so, write a model that gives y as a function of x .

5.

x	2	4	6	8	10
y	18	9	6	4.5	3.6

yes; $y = \frac{36}{x}$

JOINT VARIATION

Joint variation occurs when a quantity varies directly with the product of **two or more** other quantities. In the equation below, a is a nonzero constant.

$$z = \underline{axy}$$

z varies jointly with x and y .

Example 4 Write a joint variation equation

The variable z varies jointly with x and y . Also, $z = -84$ when $x = -4$ and $y = 3$. Write an equation that relates x , y , and z . Find z when $x = 5$ and $y = 2$.

Write the general joint variation equation. Use the given values of z , x , and y to find the constant of variation a .

$$z = axy$$

$$\underline{-84} = a(\underline{-4})(\underline{3}) \quad \text{Substitute for } z, x, \text{ and } y.$$

$$\underline{-84} = \underline{-12}a \quad \text{Simplify.}$$

$$\underline{7} = a \quad \text{Solve for } a.$$

The joint variation equation is $z = \underline{7xy}$. Calculate z when $x = 5$ and $y = 2$ using substitution.

$$z = \underline{7xy} = \underline{7(5)(2)} = \underline{70}$$

Your Notes

Example 5 Compare different types of variation

Write an equation for the relationship.

Relationship	Equation
a. m varies jointly with n , p , and q .	$m = \frac{anpq}{1}$
b. r varies inversely with s .	$r = \frac{a}{s}$
c. x varies inversely with the cube of y .	$x = \frac{a}{y^3}$
d. k varies jointly with x and y and inversely with m .	$k = \frac{axy}{m}$
e. t varies directly with u and inversely with w .	$t = \frac{au}{w}$

✓ **Checkpoint** Complete the following exercises.

6. The variable z varies jointly with x and y . Also, $z = -44$ when $x = 4$ and $y = -1$. Write an equation that relates x , y , and z . Find z when $x = 6$ and $y = 3$.

$$z = 11xy; 198$$

7. Write an equation for the relationship: x varies jointly with y and z and inversely with the square of t .

$$x = \frac{ayz}{t^2}$$

Homework